

Measuring Segregation and Controlling for Independent Variables

Arbeitspapiere

Working papers

Frank Kalter

Arbeitspapiere -Mannheimer Zentrum für Europäische Sozialforschung ISSN 1437-8574

Nr. 19, 2000

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Kalter, Frank:

Measuring Segregation and Controlling for Independent Variables / Frank Kalter. – Mannheim : 2000

(Arbeitspapiere - Mannheimer Zentrum für Europäische Sozialforschung ; 19) ISSN 1437-8574

Not available in book shops. Token fee: DM 5,– Purchase: Mannheimer Zentrum für Europäische Sozialforschung (MZES), D – 68131 Mannheim WWW: http://www.mzes.uni-mannheim.de

Editorial Note:

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Abstract

This paper suggests a procedure to control for independent variables in the measurement of segregation by linking the well known Index of Dissimilarity and the Multinomial Logit Model. While the first one may be considered as a standard macro measure of inequality, the latter one is a typical tool to analyze the determinants of individual behavior and attainment. Combining both enables a judgement or a comparison of inequality structures taking into account respective distributions of relevant influential variables. After a short review of the debates on segregation indexes, the technique is described in detail. It is further illustrated using an example dealing with the assimilation by family types of foreigners in the Federal Republic of Germany. It is demonstrated that the method may be a helpful tool to come to more adequate judgements concerning the development of inequality structures.

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1. Background

Measures of segregation are indispensable tools in the analysis of social inequality, allowing us to describe complex structural patterns by one single quantity. Common fields of application are for example ethnic residential segregation, where it is necessary to measure the amount of disparity in the distribution of different ethnic groups over different residential districts, or occupational segregation (by sex or ethnicity) in the labor market, where the focus is on disparities concerning the distribution of different economic branches or occupations. The demand for a really single measure resolves from the fact that interest often centers on a comparison of inequality structures over time or between structural contexts. Typical questions are: Has residential segregation of Blacks, Hispanics, and Asians decreased between 1970 and 1980 (Massey/Denton 1987)? Are there differences of female integration into the economy between different western societies (OECD 1985)? One could say, that the main purpose of segregation measures is to capture social inequality as a characteristic of macro structures.

A quite different starting point for the analysis of social inequality is to focus on the determinants of individual behavior and attainment. The most typical methodological tool here is regression analysis in its widest sense: linear regression, logistic regression, event history analysis and so on (Brüderl 2000). The advantage of multivariate regression techniques is the possibility for a more complex judgment on different causes and their interplay on a specific aspect, for example income, upward mobility and belonging to a specific class or category. With respect to the above mentioned fields of application, typical questions now are: Which individual and contextual factors increase the probability of a black person residing in suburban district with a high percentage of whites? Which individual and contextual factors increase the probability of a female person belonging to a specific occupational group normally dominated by men? The main purpose of regression analysis is the measurement of complex impacts on social inequality as an individual or micro characteristic.

While both procedures are in some sense complementary in their advantages, they are also complementary in their disadvantages. Concerning segregation measures, the price of describing inequality parsimoniously is paid by blurring interesting details or individual differences. Concerning regression analysis, the complex picture expressed in diverse coefficients and standard errors makes it hard to come to a short summarizing description of the underlying inequality structure which may be easily compared between different contexts. As a consequence, if interests are partly in both aspects of social inequality, i.e. as well in the overall picture as in some of the individual or specific aspects, it is necessary to utilize both methods. An abstract research question would be: What are the differences and developments of inequality on the macro level *taking into account* some of the determinants of individual attainment? Therefore it is an obvious question, whether both tools have to stand separated from each other or whether there is some easy link between measures of segregation and regression analysis.

In this paper we want to propose such a link. The specific question that motivated our considerations is of the same structure as the general kind described above: If we look at different family types and take into account that these are influenced by age and sex: Is there a convergence between foreigners and natives in Germany between 1970 and 1995? To answer this question we looked for a convenient way of controlling for independent variables within segregation measures. Our proposal relies on the most common segregation measure, the Index of Dissimilarity, and a special variant of regression analysis, the Multinomial Logit Model. The whole paper is divided into three major parts. It seems necessary to start with a brief overview of the discussions on the measures of segregation, since the Index of Dissimilarity is not only the most popular index but also the most criticized one.¹ Afterwards we show, how the Index of Dissimilarity may be connected with the coefficients of the Multinomial Logit Model and how it may be computed under control of independent variables. Finally, we illustrate our procedure applying it to the question of whether family types of foreigners and German natives have converged between 1970 and 1995, taking into account, that the distributions of sex and age are different between both groups.

2. A short overview of measures of segregation: A plea for the Index of Dissimilarity

Long before and long after the classical "methodological analysis of segregation indexes" by Otis Dudley Duncan and Beverly Duncan (1955) the problem of how to measure segregation has been discussed intensively. Even today there is still no consent on the question of which measure is the most preferable one. On the one hand, this is due to the fact that for a long period of time clear criteria for the judgement of the suitability of different indexes seemed to be lacking (James/Taeuber 1985: 2) or rather that researchers were (and are) not aware of them. On the other hand, different purposes imply different criteria, so that it seems obvious that no single measure will be best suited for every kind of application (Lieberson 1980: 253; Massey/Denton 1988: 283). In the long history of the 'index debate' time after time new measures have been proposed, as the existing ones are flawed in certain respects. Soon after their appearance these are rejected themselves, because they are flawed in some other (and often more important) respect. As a consequence, researchers checking the literature trying to find help and advice for handling a specific problem in a field of application have a hard time finding state of the art solutions. In addition to that, they will become increasingly unsure, for they will soon learn, that the choice is not open. Different indexes may lead to different results (OECD 1985: 42-44; James/Taeuber 1985: 19-22; Blackburn et al. 1993: 337-341; Hakim 1993: 293-295) and there is an obvious danger of making your own analysis suspect of being arbitrary.

In spite of all the controversies and non-transparencies no one will doubt that the Index of Dissimilarity (D) stands supreme among the numerous indexes available. Surely, it is the index most frequently used and most frequently criticized. There are at least four reasons for the prominence of D: Firstly, D

¹ We also decided to spend a few pages on this review, because it is still hard to find straight advice as a practitioner in spite of the fact that many contributions are available.

has a rather simple and intuitive interpretation. Therefore, it is the natural choice in a situation, where measures are highly correlated and the conclusions drawn out of them are similar (e.g. Massey/Denton 1987). Secondly, D has a number of properties, that are desirable in the context of many applications. Thirdly, even if D shows weakness in some respects, there is no alternative measure which is not weak in other respects. Fourthly, if D is the most popular measure of segregation the process of index choice is self-enforcing, giving authors the possibility to compare their results to other analyses. This holds true at least as long as the third point is valid. These four arguments will become clear, if we briefly discuss the Index of Dissimilarity and some of the alternatives in detail.

• The Index of Dissimilarity is defined by:

$$D = \frac{1}{2} \sum_{k=1}^{J} \left| \frac{A_k}{A} - \frac{B_k}{B} \right| ,$$

where J is the number of categories, A is the number of persons belonging to group A, B is the number of persons belonging to group B, A_k is the number of persons belonging to group A and category k and B_k is the number of persons belonging to group B and category k. Verbally speaking, D is half of the sum over all categories of the absolute differences between the proportion of A and the proportion of B belonging to a certain category.

By definition, D always lies in the interval [0,1]. The maximal value of 1 is reached, if the groups A and B are distributed disjunctively over the categories. The minimal value of 0 is reached if the distributions of A and B over all categories are the same. The most common interpretation of D is that it expresses the proportion of members belonging to one of the two groups which had to move to an other category in order to achieve an equal distribution of both groups over all categories (Duncan/Duncan 1955:211; for a proof: Cortese et al. 1976: 634f). Another interpretation is that D is the maximum distance of the Lorenz curve from the first main diagonal.²

What are the main features of D? Relying on the work of Schwartz and Winship (1979) James and Taeuber (1985: 11-19) formulated four criteria for a general evaluation of segregation measures. D satisfies three of them:

- 1. D satisfies the principle of *organization equivalence* which requires that the combination of two or more categories with identical proportions of A and B into a single category should leave the measure unchanged.
- 2. D satisfies the principle of *size invariance* which requires that the measure should be unchanged, if each cell of the underlying cross table is multiplied by the same constant.

3. D also satisfies the principle of *composition invariance*. This principle states, that a proportional change of the size of one of the two groups, which leaves the distribution of this group over the categories unchanged also leaves the segregation measure unchanged. As a consequence, the measure is unaffected by variations in the composition of both groups.

D fails to fully satisfy a forth criterion:

4. The principle of *transfers* holds that the segregation measure decreases when a member of group A (or group B) moves from a category with a higher A-proportion (B-proportion) to a category with a lower A-proportion (B-proportion). D does not fully comply with this principle, as its value is unaffected by movements between two categories if both are above or both are below the average proportion concerning one group.

These criteria³ can also be used to evaluate some of the competitors of D in the segregation literature. Two very prominent examples are the so-called Sex Ratio Index, used by Catherine Hakim in her studies for the British Department of Employment (Hakim 1979; 1981), and the WE index, used by the OECD (OECD 1980; 1985). However, concerning the above criteria both indexes do worse than D.

• To define the *Sex Ratio Index* (Hakim 1979; also see: Siltanen 1990: 3; Watts 1990: 595) one first has to differentiate between categories overrepresenting (in relation to the overall proportions) members of A (A-categories) and overrepresenting members of B (B-Categories). If A_A describes the number of A-members in A-categories and T_A the total number of persons being in A-categories, and if A_B and T_B are the respective numbers for B-categories, the index is described by:⁴

$$SR_{A} = \left(\frac{A_{A}}{A}\frac{T}{T_{A}}\right) - \left(\frac{A_{B}}{A}\frac{T}{T_{B}}\right).$$

Within the first brackets we find the observed proportion of A's being in A-categories divided by the 'expected' proportion. Within the second brackets we find the respective quantities for A-members in B-categories.

Siltanen (1990: 9-21) prooves that SR neither satisfies the principle of transfers nor the principle of composition invariance. In addition, two other disadvantages can be found. On the one hand SR is not symmetrical for A and B⁵, on the other hand the range of SR is not confined to a certain interval but

 ² The Lorenz curve is obtained by sorting the categories according to the percentage of group B and then plotting the cumulative distribution of A as a function of the cumulative distribution of B (James/Taeuber 1985: 6).

³ James and Taeuber also mention another criteria which they call Lorenz criteria. It states that the segregation measure for a context X should be lower than that for context Y whenever the Lorenz curve of X is somewhere above and nowhere below the Lorenz curve of Y. This principle can be seen as a summarizing criteria, because it is satisfied if all four criteria above are satisfied (James/Taeuber 1985: 19).

⁴ The subscript A is added to SR, because the index is not symmetric and A is the reference for computation.

⁵ In contrast to that D is symmetrical.

dependent on the overall proportions of A or B. The latter can be corrected by multiplying A/T to SR_A to standardize the index to values between 0 and 1 (Siltanen 1990: 12). However, the main problem, the lack of composition invariance remains (Watts 1990: 597).

• The *WE Index* received its name from the study ,Women and Employment' conducted by the OECD (OECD 1980) and is computed via:

$$WE_A = \sum_{k=1}^{J} \left| \frac{A_k}{A} - \frac{T_k}{T} \right|.$$

It can be shown that $WE_A = D \cdot 2B/T$, which means the WE_A index is twice the Index of Dissimilarity multiplied by the overall proportion of B (Blackburn et al. 1993: 344). This formulation clearly shows, that WE on the one hand also fails to satisfy the principle of transfers and on the other hand obviously is not invariant to the composition of A and B. Like the Sex Ratio Index, WE also suffers from the fact that the range of values is dependent on the group proportions and that it is not symmetric.

While the Sex Ratio Index and the WE Index seem to confront even more problems than D, James and Taeuber suggest two measures which satisfy all four criteria, these are the Gini Index and the Atkinson Index.

• Using the notations above the *Gini Index* (Duncan/Duncan 1955; James/Taeuber 1985; White 1986) is given by:

$$G = \frac{1}{2AB} \sum_{k=1}^{J} \sum_{l=1}^{J} T_{k} T_{l} \left| \frac{A_{k}}{T_{k}} - \frac{A_{l}}{T_{l}} \right|,$$

The Gini Index can be interpreted as being the fraction of the area between the Lorenz curve and the diagonal with respect to the total area under the diagonal. Therefore, there is a close connection to D which is also related to the Lorenz curve as mentioned above.

• The formal expression for the *Atkinson Index* (Schwartz/Winship 1979; James/Taeuber 1985; White 1986) is:

$$AI_{d} = \frac{A/T}{1 - A/T} \left[\sum_{k=1}^{J} \frac{(1 - \frac{A_{k}}{T_{k}})^{1 - d} (\frac{A_{k}}{T_{k}})^{d} T_{k}}{A} \right]^{1/(1 - d)}$$

It is more correct to speak of a family of indexes, because for each $\delta \in [0,1[Al_{\delta}]$ defines a different measure of segregation. The smaller the parameter δ the more it reduces the measure if desegregative transfers are made in categories overrepresented by B. Higher values of δ make

the index more sensitive to categories overrepresenting A, while a value of 0.5 places the same weight to both types of categories (James/Taeuber 1985: 22f).

It is true that the Gini Index and the Atkinson Indexes satisfy all four criteria (James/Taeuber 1985: 19) but this does not imply that they are to be preferred to D in any case. Both measures also have some drawbacks. For example, White (1986: 203-4) assumes that the Gini-Index is seldom used because its computation needs more effort. The Atkinson indexes for their part are asymmetrical, the dependence on the parameter δ makes comparisons more difficult, and under certain conditions the choice of δ may affect rankings (James/Taeuber 1985: 23-24). The main reason however, why both the Gini Index and the Atkinson family may be disadvantageous to the Index of Dissimilarity is that they do not have a comparably easy and intuitive interpretation. ⁶ As a consequence, the question for practical users is, whether in a special field of application the pros of the transfer principle outweighs the cons, especially the fact of more difficult interpretation. While the transfer principle seems indispensable in economic inequality and welfare approaches it may be irrelevant or even undesirable for segregation purposes (Blackburn et al. 1994: 416; James/Taeuber 1985: 25).

However, lacking the principle of transfers is not the only source of criticism directed against D. On the contrary, the major objections seem to refer to rather different aspects. Two specific points are mentioned repeatedly in the literature: Firstly, the Index of Dissimilarity is said not to be composition invariant. Secondly, it is said to be influenced by the sizes of the categories (e.g. Blackburn et al. 1993: 345; Cortese et al. 1976: 631; Duncan/Duncan 1955: 216; Hakim 1993: 294; 1996: 69; Massey 1977: 587; Taeuber/Taeuber 1965: 231-235).

The first statement that D is not invariant to the composition of both subgroups may sound totally surprising, because we argued above that the principle of composition invariance is fulfilled by D and therefore the opposite is true. Taking a closer look, it becomes obvious that different things are meant by the term 'composition invariance', and that the criticism raised under this label is directed at two separate aspects. The first line of reasoning can be understood as an implicit criticism of a seemingly too narrow definition of 'composition invariance'. In the sense of James and Taeuber invariance holds true only if an increase or decrease of one of the groups affects the categories exactly according to the total distribution in the initial situation. It is argued, that this is a rather unlikely case and therefore an inadequate basis for a meaningful definition of 'composition invariance' Blackburn et al. 1993: 347). While it is hard to understand this reasoning at all, it is even harder to understand, why a change which shows a different pattern of distribution over the categories should leave a segregation measure unchanged. At least one would like to know more about which patterns of change should lead to the same rate of segregation (Watts 1994: 422). The second line of arguments deal with the fact that the composition of subgroups (like the total number of individuals) has an impact on the probability that D may show certain degrees of segregation even if the individuals are distributed randomly over the

⁶ We mentioned above that the Gini Index may be interpreted as a fraction of the area under the first main diagonal. While this is better than nothing, it is far from easy and intuitive.

categories (Cortese et al. 1976; Winship 1977). To take this into account, it is proposed that a random segregation be chosen as a reference for an index rather than zero segregation.

• For example, one could think of a Standard Score of D (Cortese et al. 1976: 633):

$$Z = \frac{D - \boldsymbol{m}_D}{\boldsymbol{s}_D}.$$

The Standard Score of D is built by subdtracting the randomly expected value of D (μ_D) from the observed value and dividing this difference by the respective standard deviation (σ_D).

However, Massey (1977: 587) objects that this standard score is itself dependent on the proportion of subgroups and therefore can not remove the problem. As a consequence, he proposes to keep D as the measure and use Z as a rough test of the hypothesis that the observed amount of segregation could be a result of pure chance (Massey 1977: 588).

We will now turn to the second statement above, D's dependence on the sizes of categories. It is criticized that D may change its value even if the proportion of both groups remains constant within each of the categories. For example, this would be the case if one doubles the persons in one category leaving all other categories unchanged. A more general formulation of the problem is that D places more weight on categories with high frequencies. To avoid this, another type of standardization has been developed:

• The *Standardized Index of Dissimilarity* was proposed by Gibbs (1965) and may be directly computed by (Charles/Grusky 1995: 935):

$$DST = \frac{1}{2} \sum_{k=1}^{J} \left| \frac{\frac{A_{k}}{T_{k}}}{\sum_{l=1}^{J} \frac{A_{l}}{T_{l}}} - \frac{\frac{B_{k}}{T_{k}}}{\sum_{l=1}^{J} \frac{B_{l}}{T_{l}}} \right|$$

The general idea of the standardization is to simulate a situation where the proportions of both groups correspond to the observed proportions in each of the categories and where all categories have equal sizes.

The Standardized Index of Dissimilarity eliminates the impact of varying category sizes in a very radical way, and it is open to question whether this is a desirable procedure. Should categories with only few cell counts really have the same impact on an index as those with noticeable shares of the total population? Should the index really stay unchanged if for example 'fair' categories gain or loose weight? It soon becomes clear that such a standardization blurs interesting aspects of segregation and desegregation and therefore cannot be a patent remedy to the problem of measuring it.

It seems to make more sense to admit that the rate of segregation may be influenced as well by changes of the relative access of subgroups to certain categories as well as by structural changes concerning the development of categories (decreasing or increasing tendencies leave the structure of relative access unchanged). While DST simply eliminates the latter cause, the problem of D is that it is a correct measure of both kinds of changes together. However, sometimes the interest lies in only one of the two components of change or in the relative extent of each. Instead of searching for new indexes which totally neglect one of the two effects, a more natural solution would be to try to separate them within D. In the meantime some suggestions are available which allow for a decomposition of a temporal change of D into both basic processes (sometimes an additional 'mixed' effect is added) (Blau/Hendricks 1979; Handl 1984). It may even be interesting to analyze how a third development, changes in the composition of both groups, may affect trends in segregation. Logically, an index like D, which is composition invariant, cannot account for such changes. Karmel and McLachlan (1988) therefore propose a new index, which is closely related to D:

• The *IP Index* may be directly computed by (Karmel/McLachlan 1988: 188)

$$IP = \frac{1}{T} \sum_{k=1}^{J} \left| \frac{B}{T} A_k - \frac{A}{T} B_k \right|$$

or simply be derived from D via (Watts 1992: 480):

$$IP = 2\frac{A}{T}\frac{B}{T}D.$$

While D can be interpreted as the fraction of one of the groups which would have to relocate in order to get zero segregation, IP can be interpreted as the corresponding fraction of the total population. IP may be used for decomposition of changes in segregation into three basic elements (Karmel/McLachlan 1988, Watts 1992).

This section should have shown that the Index of Dissimilarity has a number of convenient features and that a number of objections are either irrelevant or unfounded. Further, there is no alternative which does a better job simultaneously in all or even most respects and many other indexes and additional procedures are build around D, so that a wide range of tools is available if it is chosen as the standard measure for segregation. In the following section we will add another argument in favor of D resolving from the fact, that D may be computed only using the relative frequencies of category membership for each of the subgroups. As we shall see, this characteristic opens the door for a convenient inclusion of control variables using the Multinomial Logit Model.

3. Combining the Index of Dissimilarity and the Multinomial Logit Model

The link between the Index of Dissimilarity and the Multinomial Logit Model rests on the conditional probabilities of belonging to each of the given categories dependent on group membership. While D can be computed from these conditional probabilities, the Multinomial Logit Model allows one to regress them on a set of independent variables. We will now describe this basic idea in detail.

The Multinomial Logit Model is the extension of the logistic regression model to dependent variables with J nominal outcomes. In its general form the probability of an actor i belonging to category j is given by

$$\Pr(y_i = j \mid x_i) = \frac{\exp(x_i \boldsymbol{b}_j)}{\sum_{k=1}^{J} \exp(x_i \boldsymbol{b}_k)},$$

where x_i is a vector containing the values of m covariates for person i and β_k is a vector of m+1 parameters β_{0k} , β_{1k} , , β_{mk}) for each k = 1, ..., J (e.g. Long 1997: 152). In order to identify the parameters it is common to choose one reference category and set the corresponding vector of parameters equal to a vector of zeroes.

	group A	group B
	$x_1 = 0$	x ₁ = 1
Y=1	$\frac{\exp(\boldsymbol{b}_{01})}{1+\sum_{k=1}^{J-1}\exp(\boldsymbol{b}_{0k})}$	$\frac{\exp(\boldsymbol{b}_{01} + \boldsymbol{b}_{11})}{1 + \sum_{k=1}^{J-1} \exp(\boldsymbol{b}_{0k} + \boldsymbol{b}_{1k})}$
Y = j	$\frac{\exp(\boldsymbol{b}_{0j})}{1+\sum_{k=1}^{J-1}\exp(\boldsymbol{b}_{0k})}$	$\frac{\exp(\boldsymbol{b}_{0j} + \boldsymbol{b}_{1j})}{1 + \sum_{k=1}^{J-1} \exp(\boldsymbol{b}_{0k} + \boldsymbol{b}_{1k})}$
Y= J-1	$\frac{\exp(\boldsymbol{b}_{0(J-1)})}{1+\sum_{k=1}^{J-1}\exp(\boldsymbol{b}_{0k})}$	$\frac{\exp(\boldsymbol{b}_{0(J-1)} + \boldsymbol{b}_{1(J-1)})}{1 + \sum_{k=1}^{J-1} \exp(\boldsymbol{b}_{0k} + \boldsymbol{b}_{1k})}$
Y = J	$\frac{1}{1+\sum_{k=1}^{J-1}\exp(\boldsymbol{b}_{0k})}$	$\frac{1}{1+\sum_{k=1}^{J-1}\exp(\boldsymbol{b}_{0k}+\boldsymbol{b}_{1k})}$

Table 1: Column percentages of a J² table expressed by the Multinomial Logit Model

A convenient feature of the Multinomial Logit Model is the possibility to reproduce the column percentages of a J_2 cross table. If one chooses the variable containing the J categories as the

dependent (with J being the reference category) and a dummy variable x_1 for group membership ($x_{1i} = 0$ for all i belonging to A and $x_{1i} = 1$ for all i belonging to B) as the only independent variable, the column percentages may be expressed as in table 1.

Since the Index of Dissimilarity can be computed out of the column percentages, it can also be derived from the estimates of a Multinomial Logit Model. According to the definition of D and to table 1 we find that:

$$D = \frac{1}{2} \left(\sum_{k=1}^{J-1} \left| \frac{\exp(\mathbf{b}_{0k})}{1 + \sum_{l=1}^{J-1} \exp(\mathbf{b}_{0l})} - \frac{\exp(\mathbf{b}_{0k} + \mathbf{b}_{1k})}{1 + \sum_{l=1}^{J-1} \exp(\mathbf{b}_{0l} + \mathbf{b}_{1l})} \right| + \left| \frac{1}{1 + \sum_{l=1}^{J-1} \exp(\mathbf{b}_{0l})} - \frac{1}{1 + \sum_{l=1}^{J-1} \exp(\mathbf{b}_{0l} + \mathbf{b}_{1l})} \right| \right)$$

As it is easy to see, we would also obtain D if we apply its standard definition (see page 3) to the following cross-table (table 2)

k	A _k	B _k
1	$exp(\beta_{01})$	$exp(\beta_{01}) \cdot exp(\beta_{11})$
j	exp(β _{0j})	$exp(\beta_{0j}) \cdot exp(\beta_{1j})$
J-1	$exp(\beta_{0(J-1)})$	$exp(\beta_{0(J-1)}) \cdot exp(\beta_{1(J-1)})$
J	1	1

Table 2: An odds table as a starting point for the computation of D

It is a useful property of the Multinomial Logit Model that it may be interpreted in terms of an odds model (Hosmer/Lemeshow 1989: 220-225; Long 1997: 154). In our case, the elements contained in the cells of table 2 are the odds of a member of the respective column belonging to the category of the respective row versus the reference category J. For each k the odds of the members of A are equal to $exp(\beta_{0k})$. The odds of members of B are equal to the odds of A multiplied by the so-called odds ratio $exp(\beta_{1k})$. The exponentiation of β_{1k} yields the ratio of the odds of a B-person belonging to category k versus category J. In this

simple case with one independent dummy variable for group membership these odds ratios are identical with those obtained from the underlying cross-table.⁷

As shown above, it is possible to include further independent variables into the model. Let us assume that we consider m-1 additional variables $x_2,...,x_m$ which leads to an estimate of m+1 parameters (β'_{0k} , β'_{1k} , β'_{2k} , β'_{mk}) for each k = 1, ..., J. The exponentiation of β'_{1k} now is still interpretable in terms of an odds ratio, but it is not the overall odds ratio resulting from the cross table but the odds ratio holding all other variables constant (Long 1997: 154). The expression $\exp(\beta'_{1k})$ may be seen as the factor by which the odds of a member of A must be multiplied in order to get the odds of a member of B, assuming that both have the same values for all $x_2, ..., x_m$.

In order to control for independent variables within D, our proposal now is to use these 'controlled' odds ratios instead of the overall odds ratios, or more precisely to compute an adjusted Index of Dissimilarity D' from:

k	A _k	B _k
1	$exp(\beta_{01})$	$exp(\beta_{01}) \cdot exp(\beta'_{11})$
j	$exp(\beta_{0j})$	$exp(\beta_{0j}) \cdot exp(\beta'_{1j})$
J-1	$exp(\beta_{0(J-1)})$	$exp(\beta_{0(J-1)}) \cdot exp(\beta'_{1(J-1)})$
J	1	1

Table 3:	An odds table as a starting point for the computation of D'
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We still use β_{0k} instead of β'_{0k} because this reflects the 'mean' covariate constellation of $x_2, ..., x_m$, which seems more appropriate for our purposes than a constellation with the reference value for each covariate. As a result, we get the following definition:

• The Adjusted Index of Dissimilarity D', which is 'holding constant the variables x₂, ..., x_m', may be computed by

⁷ The odds ratios also play an important part in the margin-free measure of segregation recently proposed by Charles and Grusky (1995).

$$D' = \frac{1}{2} \left(\sum_{k=1}^{J-1} \left| \frac{\exp(\boldsymbol{b}_{0k})}{1 + \sum_{l=1}^{J-1} \exp(\boldsymbol{b}_{0l})} - \frac{\exp(\boldsymbol{b}_{0k} + \boldsymbol{b'}_{1k})}{1 + \sum_{l=1}^{J-1} \exp(\boldsymbol{b}_{0l} + \boldsymbol{b'}_{1l})} \right| + \left| \frac{1}{1 + \sum_{l=1}^{J-1} \exp(\boldsymbol{b}_{0l})} - \frac{1}{1 + \sum_{l=1}^{J-1} \exp(\boldsymbol{b}_{0l} + \boldsymbol{b'}_{1l})} \right| \right)$$

where β_{0k} are the constants of a Multinomial Logit Model containing only a group membership dummy x_1 and β'_{1k} are the coefficients of x_1 in a model also containing independent variables x_2 , ..., x_m .

4. Example: Assimilation of foreigners by family types in Germany between 1970 and 1995

We now want to illustrate the proposed procedure by applying it to the question of whether there is a convergence of family types between foreigners and natives in Germany between 1970 and 1995. To analyze the situation in both years we use a 1% sample of the Population and Occupation Census 1970 and the 70% sample (ZUMA-File) of the Microscensus 1995, which itself is a 1% sample of the population in Germany.⁸ Many variables are rather similar in both data sets, because the underlying questions and categories are completely or nearly the same. In our case we can derive the same family typology from both censuses. By family type we mean the kind of family the respective person is actually living in. We distinguish nine types, which are shown in the first column of table 4. In addition to that the table shows the absolute numbers and the column percentages for Germans and foreigners falling in each of the categories in both years.

If we compute the Index of Dissimilarity for the two subtables we get a value of 0,195 for 1970 and 0,206 for 1995. According to these two figures the conclusion would be that there has not been a convergence of family types within those 25 years but rather a slight divergence.

Obviously, this conclusion seems to be drawn too fast, because during that period a lot of structural changes have occurred in Germany and it is reasonable to assume that some of them have influenced the distribution of family types between the two subgroups. One of the things which lie near at hand is that due to the historical development of labor migration into Germany (recruitment in the 60's, afterwards processes of family reunion) the age and sex structure of foreigners as well as of Germans has fundamentally changed. The extent of this demographic change is shown in table 5.

⁸ We excluded the persons living in regions of Germany formerly belonging to the GDR to make the results more comparable to 1970.

	1970 1995		95	
	Germans	foreigners	Germans	foreigners
1 married couple, without children	101675	3565	88565	4159
	17,5%	17,6%	23,4%	13,3%
2 married couple with child(ren)	182270	5134	94766	11010
	31,3%	25,4%	25,1%	35,3%
3 divorced or widowed without children	48391	617	39465	1000
	8,3%	3,0%	10,4%	3,2%
4 divorced or widowed with child(ren)	12544	138	7996	486
	2,2%	0,7%	2,1%	1,6%
5 never married with child(ren)	1331	49	2276	159
	0,2%	0,2%	0,6%	0,5%
6 married, separated without children	5725	2012	3729	763
	1,0%	9,9%	1,0%	2,4%
7 married, separated with child(ren)	1474	174	1378	192
	0,3%	0,9%	0,4%	0,6%
8 never married, no children, not living	27137	2919	41906	2652
with parents	4,7%	14,4%	11,1%	8,5%
9 never married, no children, living with	201393	5641	97827	10741
at least one of the parents	34,6%	27,9%	25,9%	34,5%
total	581940	20249	377908	31162
	100,0%	100,0%	100,0%	100,0%
D		195		206

Table 4:Family types of Germans and foreigners in 1970 and 1995 (absolute numbers and
column percentages)

Table 5:Age and sex structure of Germans and foreigners in 1970 and 1995 (column
percentages)

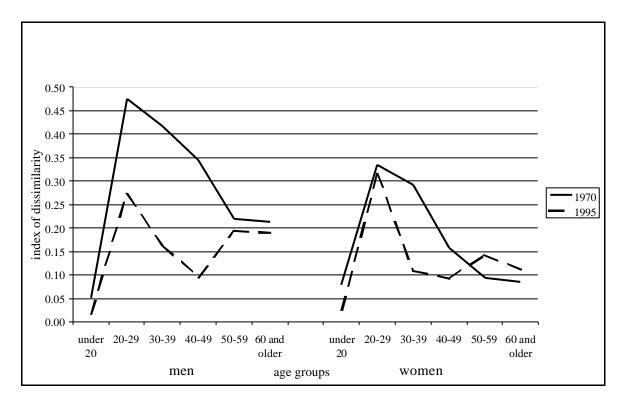
	Germans		forei	igners
	1970	1995	1970	1995
Men, under 20 years old	15.0%	10.2%	11.4%	16.1%
Men, 20 to 29 years old	6.7%	6.9%	17.6%	10.4%
Men, 30 to 39 years old	7.2%	7.9%	21.0%	9.3%
Men, 40 to 49 years old	5.8%	6.4%	8.8%	7.3%
Men, 50 to 59 years old	4.4%	7.4%	2.8%	6.6%
Men, 60 years and older	8.1%	9.3%	1.6%	2.7%
Women, under 20 years old	14.3%	9.7%	11.3%	14.6%
Women, 20 to 29 years old	6.3%	6.7%	11.3%	10.4%
Women, 30 to 39 years old	7.0%	7.7%	7.4%	8.1%
Women, 40 to 49 years old	6.9%	6.4%	3.7%	7.7%
Women, 50 to 59 years old	6.1%	7.5%	1.4%	4.5%
Women, 60 years and older	12.3%	13.9%	1.5%	2.1%
D: Germans 1970 – foreigners 1970	0,	333		
Germans 1995 – foreigners 1995	0,	221		
Germans 1970 – Germans 1995	0,	099		
foreigners 1970 – foreigners 1995	0,	213		

The age and sex distribution was very different for Germans and foreigners in 1970. The latter group was highly overrepresented in the categories for men between 20 and 39 and in the category of women between 20 and 29. If we express the age-sex inequality of Germans and foreigners in terms

of D, we get a value of 0,333 for 1970. This discrepancy has been reduced in the years up to 1995 but is still noticeable; it now reaches a value of 0,221. As we can see in the table, the distribution of both groups changed within that period of time. We notice that there is a slight shift for Germans (D = 0,099) and a strong shift for foreigners (D = 0,213). Compared to Germans foreigners in 1995 are overrepresented in the younger age groups and underrepresented in the older ones, especially in the group of older women.

Naturally, the choice of family type is closely related to sex and age. If we want to control for the distribution of these two variables one possible procedure is to compute the dissimilarity in family types for each sex-age group separatly. Figure 1 represents the corresponding pattern graphically. As a result we get the Index of Dissimilarity with respect to family type for each age group of men and women in 1970 and 1995.

Figure 1: Dissimilarity in family types in 1970 and 1995 computed separately for sexes and age groups



Compared to the overall index values in table 4 figure 1 gives a different answer to the question of whether there has been an assimilation in family styles between natives and foreigners since 1970. Except for the two oldest age groups of women the Index of Dissimilarity has dropped for each subgroup in this 25 years. The trend towards assimilation of family types is most remarkable for men in the middle age range. We also see that in 1970 as well as in 1995 the differences between

Germans and foreigners are strongest in the age group 20-29 years, which holds true for men as well as for women.

Figure 1 gives a more correct picture of the family type assimilation, because the separated calculation for age groups and sexes controls for the respective structural shifts. A disadvantage of this representation is that the information included is rather detailed. If we want to control for further variables it will be hard to come to a clear picture and we will soon reach the limits of our sample size. Therefore we now want to apply the procedure outlined in the above section to come to a more condensed judgment about family type assimilation, which also controls for the changes in age and sex structure.

First of all, we compute a Multinomial Logit Model choosing category 9 of the family typology (single, no children, living with at least one of the parents) as the reference category and including only one dummy for being a foreigner (Germans = 0) as an independent variable in the model. We used the method of 'individualized regressions' Begg/Gray 1984; Hosmer/Lemeshow 1989) to estimate the parameters because the maximum-likelihood algorithms converged faster this way.⁹ The estimated coefficients of the constant and the foreigner dummy are shown in the columns of table 6 for 1970 and 1995.

	coefficie	coefficients for 1970		coefficients for 1995	
k	constant	foreigner	constant	foreigner	
	β _{0k}	β_{1k}	β _{0k}	β_{1k}	
1 (married couple, without children)	-0.68348	0.224580	-0.099464	-0.849330	
2 (married couple with child(ren))	-0.09977	0.005593	-0.031790	0.056526	
3 (divorced or widowed without children)	-1.42594	-0.787003	-0.907786	-1.466282	
4 (divorced or widowed with child(ren))	-2.77602	-0.934547	-2.504259	-0.591356	
5 (never married, with child(ren))	-5.01933	0.273331	-3.760781	-0.452138	
6 (married, separated without children)	-3.56042	2.529484	-3.267061	0.622495	
7 (married, separated with child(ren))	-4.91728	1.438517	-4.262567	0.238239	
8 (never mar., no child, n. liv. w. parents)	-2.00436	1.345540	-0.847772	-0.550983	
9 (never mar., no child, living w. parents)	0	0	0	0	

Table 6: Results of estimating a Multinomial Logit Model with a dummy for foreigners

We may now use the estimated coefficients to reproduce the column percentages according to table 1. As a result we get table 7 which reflects the situation in table 4 and leads to the same values for the Index of Dissimilarity.

⁹ Due to the the sample size this was an important feature even using the computer power of the 21st century. For conducting the method of ,individualized regressions' one creates J-1 dummy variables (if there are J categories of the dependent variable) and sets them equal to one, if an individual belongs to the respective category, equal to zero, if an individual belongs to the reference category, and treats it as missing, if an individual belongs to neither of both. After that one estimates J-1 (binary) logistic regressions using each of the J-1 dummy as a dependent variable.

	1970		19	995
k	$\exp(\boldsymbol{b}_{0k})$	$\exp(\boldsymbol{b}_{0k} + \boldsymbol{b}_{1k})$	$\exp(\boldsymbol{b}_{0k})$	$\exp(\boldsymbol{b}_{0k} + \boldsymbol{b}_{1k})$
	$\sum_{j=1}^{J} \exp(\boldsymbol{b}_{0i})$	$\sum_{j=1}^{J} \exp(\boldsymbol{b}_{0j} + \boldsymbol{b}_{1j})$	$\sum_{j=1}^{J} \exp(\boldsymbol{b}_{0j})$	$\sum_{i=1}^{J} \exp(\boldsymbol{b}_{0i} + \boldsymbol{b}_{1i})$
	$\sum_{l=1}^{l} r < 0$	$\sum_{l=1}^{l} r < 0$	$\sum_{l=1}^{l}$	$\sum_{l=1}^{l} r < 0 $
1 (married couple, without children)	0.174717	0.176058	0.234356	0.133464
2 (married couple with child(ren))	0.313211	0.253543	0.250765	0.353315
3 (divorced or widowed without children)	0.083155	0.030471	0.104430	0.032090
4 (divorced or widowed with child(ren))	0.021555	0.006815	0.021159	0.015596
5 (never married, with child(ren))	0.002287	0.002420	0.006023	0.005102
6 (married, separated without children)	0.009838	0.099363	0.009867	0.024485
7 (married, separated with child(ren))	0.002533	0.008593	0.003646	0.006161
8 (never mar., no child, n. liv. w. parents)	0.046632	0.144155	0.110889	0.085104
9 (never mar., no child, living w. parents)	0.346072	0.278582	0.258865	0.344683
D	0,	195	0,2	206

Table 7: Transformation of coefficients from table 6 to compute the Index of Dissimilarity

The next step in our procedure is to control for the combinations of sex and age categories. Choosing 'men, under 20 years old' as the reference group we include 11 additional dummy variables in our Multinomial Logit Model to control for the 12 subgroups shown in table 5. Table 8 contains the estimated coefficients for the foreigner dummy in 1970 and 1995.

Table 8:Coefficients of foreigner dummy in estimating a Multinomial Logit Model including
also dummies for sex and age groups

k	1970	1995
	β' _{1k}	β' _{1k}
1 (married couple, without children)	2.162624	1.217725
2 (married couple with child(ren))	1.091510	1.456424
3 (divorced or widowed without children)	1.921613	1.014277
4 (divorced or widowed with child(ren))	0.958282	0.979550
5 (never married, with child(ren))	1.606587	0.319284
6 (married, separated without children)	3.406144	2.065115
7 (married, separated with child(ren))	2.521687	1.324524
8 (never mar., no child, n. liv. w. parents)	2.218116	0.240488
9 (never mar., no child, living w. parents)	0	0

If we compare those estimates with the 'unadjusted' estimates in table 6, we notice some remarkable changes. Most clearly, in 1970 and 1995 nearly all parameters increase, which means that the reference category 'never married, no child, living with at least one parent' is overestimated for foreigners not taking into account the different demographic distributions. If we use the adjusted coefficients to derive the adjusted Index of Dissimilarity, we get the picture outlined in table 9.

	1970		1970 1995	
k	$\exp(\boldsymbol{b}_{0k})$	$\exp(\boldsymbol{b}_{0k} + \boldsymbol{b'}_{1k})$	$\exp(\boldsymbol{b}_{0k})$	$\exp(\boldsymbol{b}_{0k} + \boldsymbol{b}'_{1k})$
	$\sum_{\alpha}^{J} \exp(\boldsymbol{b}_{\alpha})$	$\sum_{l=1}^{J} \exp(\boldsymbol{b}_{0l} + \boldsymbol{b}'_{1l})$	$\sum_{\alpha}^{J} \exp(\boldsymbol{b}_{\alpha})$	$\sum_{l=1}^{J} \exp(\boldsymbol{b}_{0l} + \boldsymbol{b}'_{1l})$
	l=1	l=1	l=1	$\sum_{l=1}^{l} r < 0l$
A (manufacture la south as the laboration	0 474747	0.000400	0.004050	0.0000.40
1 (married couple, without children)	0.174717	0.362488	0.234356	0.292042
2 (married couple with child(ren))	0.313211	0.222646	0.250765	0.396736
3 (divorced or widowed without children)	0.083155	0.135573	0.104430	0.106179
4 (divorced or widowed with child(ren))	0.021555	0.013411	0.021159	0.020779
5 (never married, with child(ren))	0.002287	0.002721	0.006023	0.003056
6 (married, separated without children)	0.009838	0.070780	0.009867	0.028694
7 (married, separated with child(ren))	0.002533	0.007525	0.003646	0.005056
8 (never mar., no child, n. liv. w. parents)	0.046632	0.102268	0.110889	0.052006
9 (never mar., no child, living w. parents)	0.346072	0.082586	0.258865	0.095453
adjusted D	0	,362	C	,226

Table 9:	Combination of coefficients from table 6 and table 8 to compute the adjusted Index
	of Dissimilarity

While the first columns for 1970 and 1995 still show the column percentages for the German group, the second column for each year now shows the 'adjusted column percentages of foreigners, if the distribution of Germans and foreigners should be identical over the sex-age-groups'. Compared to the situation in table 4 or table 7 we find for example, that the percentage of the categories 1 and 3 would strongly increase for 1970 and 1995, and that the percentage of category 9 would strongly decrease in both years. While these are the most obvious changes, other categories are also affected by interesting shiftings. However, the most interesting information contained in table 9 is the value of the adjusted Index of Dissimilarity for both years, which – compared to the respective value in table 7 – reflects in some sense a summarization of all corrections. Now, we find that the value for 1970 has remarkably increased to 0,362 (compared to 0,195) while the value for 1995 has only slightly changed to 0,226 (compared to 0,206).

The temporal trend shown in these figures is in line with the trend in figure 1. The conclusion now is quiet different than that drawn from the crude situation in table 3: If we take into account that Germans and foreigners living in Germany had and still have different demographic distributions, we find that their family types noticeably converged within a quarter of a century since 1970.

5. Final remarks

It is by no means an exaggeration to state that the Index of Dissimilarity is the most prominent and most frequently used measure of segregation. But, in spite of this fact it is also true that it has been the target of much critism within the decades since the seminal work of Otis Dudley Duncan and Beverly Duncan (1955). While some of the objections are either unfounded or of minor importance, a more serious problem seems to arise from the fact, that the index may be affected by structural conditions. Due to this fact, comparisons between contexts or between different time points are more

difficult or sometimes even impossible. In the literature we find two different strategies to deal with this problem. On the one hand researchers develop and propose new indexes, but until now no alternative has been broadly accepted as a convincing solution. This is not surprising, for the purposes and research interests are very difficult and the pros and cons of the indexes weigh differently in different kinds of applications. On the other hand some attempts have been made to solve the problem without discharging the Index of Dissimilarity completely, thus conserving its advantages. For example, there have been important advances in decomposing the changes over time into structural changes and changes of relative access (e.g. Blau/Hendricks 1979; Handl 1984; Karmel/McLachlan 1988; Watts 1992).

The procedure proposed in this paper is in line with the latter strategy, but in contrast to the previous work it focusses on structural changes concerning 'independent' variables rather than changes in the distribution of the (dependent) variable at interest itself. However, it seems worth noting that the MNML-approach could also be used to decompositions concerning structural changes in the dependent variable.¹⁰ Therefore the approach seems to be a very general one, enabling also comparisons between different contexts, if the underlying distributions of relevant characteristics are very different. All in all, it delivers a fruitful method for an analysis of (macro) inequality structures, taking into account contextual and temporal differences in relevant (micro) determinants.

¹⁰ For example, if we compute Multinomial Logit Models for two different time points containing only dummies for group membership, we will receive coefficient vectors b¹₀ and b¹₁ for time 1 and b²₀ and b²₁ for time 2. If we consider b¹₀ and a combination of b¹₀ and b²₁ in an odds table like table 3, we may compute a value of D assuming a distribution of the dependent variable like at time 1 and an access structure like at time 2, thus following the idea of Handl (1984: 340).

References

- Begg, C.B./ Gray, R. 1984: Calculation of Polychotomous Logistic Regression Parameters Using Individualized Regressions. Biometrika 71: 11–18.
- Blackburn, Robert M./ Jarman, Jennifer/ Siltanen, Janet, 1993: The Analysis of Occupational Gender Segregation over Time and Place: Considerations of Measurement and Some New Evidence. Work, Employment & Society 7: 335–362.
- Blackburn, Robert M./ Jarman, Jennifer/ Siltanen, Janet, 1994: A Reply to Lampard. Work, Employment & Society 8: 413–419.
- Blau, Francine D./ Hendricks, Wallace E., 1979: Occupational Segregation by Sex: Trends and Prospects. Journal of Human Resources 14: 197–210.
- Brüderl, Josef, 2000: Regressionsverfahren in der Bevölkerungswissenschaft. In: Diekmann, Andreas et al. (ed.), Handbuch der Bevölkerungswissenschaft. Berlin: Springer im Erscheinen
- Charles, Maria/ Grusky, David B., 1995: Models for Describing the Underlying Structure of Sex Segregation. American Journal of Sociology 100: 931–971.
- Cortese, Charles F./ Falk, R. Frank/ Cohen, Jack K., 1976: Further Considerations on the Methodological Analysis of Segregation Indices. American Sociological Review 41: 630–637.
- Duncan, Otis Dudley/ Duncan, Beverly, 1955: A Methodological Analysis of Segregation Indexes. American Sociological Review 20: 210–217.
- Gibbs, Jack P., 1965: Occupational Differentiation of Negroes and Whites in the United States. Social Forces 44: 159–165.
- Handl, Johann, 1984: Chancengleichheit und Segregation: Ein Vorschlag zur Messung ungleicher Chancenstrukturen und ihrer zeitlichen Entwicklung. Zeitschrift für Soziologie 13: 328–345.
- Hakim, Catherine, 1979: Occupational Segregation: A Study of the Seperation of Men and Women's Work in Britain, the United States and Other Countries. Research Paper No. 9. London: Department of Employment.
- Hakim, Catherine, 1981: Job Segregation: Trends in the 1970s. Employment Gazette 89: 521–529.
- Hakim, Catherine, 1993: Segregated and Integrated Occupations: A New Approach to Analysing Social Change. European Sociological Review 9: 289–314.
- Hakim, Catherine, 1996: Theoretical and Measurement Issues in the Analysis of Occupational Segregation. Pp. 67–88 in: Beckmann, Petra (ed.), Gender Specific Occupational Segregation. Beiträge zur Arbeitsmarkt- und Berufsforschung. Nürnberg: Institut für Arbeitsmarkt- und Berufsforschung der Bundesanstalt für Arbeit.
- Hosmer, David W./Lemeshow, Stanley, 1989: Applied Logistic Regression. New York: Wiley.
- James, David R./ Taeuber, Karl E., 1985: Measures of Segregation. Pp. 1–32 in: Tuma, N. B. (ed.), Sociological Methodology. San Francisco: Jossey Bass.
- Karmel, T./ MacLachlan M., 1988: Occupational Sex Segregation Increasing or Decreasing? The Economic Record 64: 187–195.
- Lieberson, Stanley, 1980: A Piece of the Pie: Black and White Immigrants Since 1880. Berkeley and Los Angeles: University of California Press.

- Long, J. Scott, 1997: Regression Models for Categorical and Limited Dependent Variables. Advanced Quantitative Techniques in the Social Sciences Series 7. Thousand Oaks: Sage.
- Massey, Douglas S., 1977: On the Measurement of Segregation as a Random Variable. American Sociological Review 42: 587–590.
- Massey, Douglas S./ Denton, Nancy A., 1987: Trends in the Residential Segregation of Blacks, Hispanics, and Asians: 1970–1980. American Sociological Review 52: 802–825.
- Massey, Douglas S./ Denton, Nancy A., 1988: The Dimensions of Residential Segregation. Social Forces 67: 281–315.
- OECD, 1980: Women and Employment. Paris: OECD.
- OECD, 1985: The Integration of Women into the Economy. Paris: OECD.
- Schwartz, J./ Winship, C., 1979: The Welfare Approach to Measuring Inequality. S. 1–36 in: Schuessler, Karl F. (ed.), Sociological Methodology 1980. San Francisco: Jossey-Bass.
- Siltanen, Janet, 1990: Social Change and the Measurement of Occupational Segregation by Sex: An Assessment of the Sex Ratio Index. Work, Employment & Society 4: 1–29.
- Taeuber, Karl E./ Taeuber, Alma F., 1965: Negroes in Cities: Residential Segregation and Neighborhood Change. Chicago: Aldine.
- Watts, Martin, 1990: The Sex Ratio Index Revisited. Work, Employment & Society 4: 595-603.
- Watts, Martin, 1992: How Should Occupational Sex Segregation be Measured? Work, Employment & Society 3: 475–487.
- Watts, Martin, 1994: A Critique of Marginal Matching. Work, Employment & Society 8: 421-431.
- White, Michael J. 1986: Segregation and Diversity Measures in Population Distribution. Population Index 52: 198–221.
- Winship, Christopher, 1977: A Revaluation of Indexes of Residential Segregation. Social Forces 55: 1058–1066.